

ECE 3614: Introduction to Communication Systems

**Double sideband AM –
Transmitted Carrier**

Spring 2017



Analog Modulation

Double Side-band Transmitted Carrier AM (DSB-TC AM)

- Let $m(t)$ denote the analog message signal
- Typically $m(t)$ represents voice or music
- Transmitted signal:

$$s(t) = A_c \cos(2\pi f_c t) + m(t) \cos(2\pi f_c t)$$

- Message modulates the amplitude and carrier is transmitted with it
- $A_c + m(t)$ is the message envelope now
- Also called double side-band transmitted carrier AM
- Modulation index is equal to peak of sidebands to peak of the carrier:
$$\mu = \frac{\max|m(t)|}{A_c} \quad \mu = \frac{m_p}{A_c} \text{ if } -m_p \leq m(t) \leq m_p$$
- We must have $0 \leq \mu \leq 1$ otherwise, the signal is over-modulated

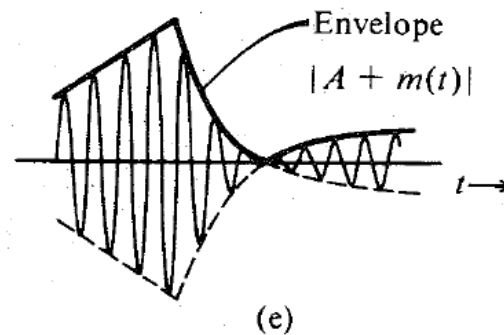
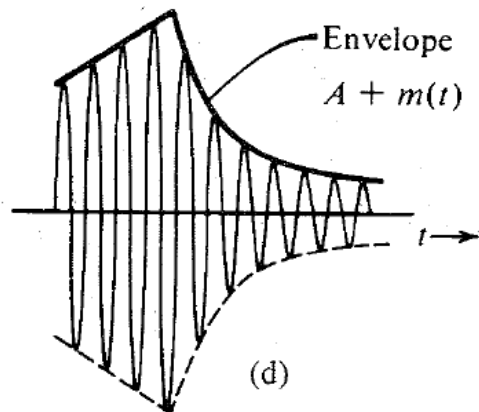
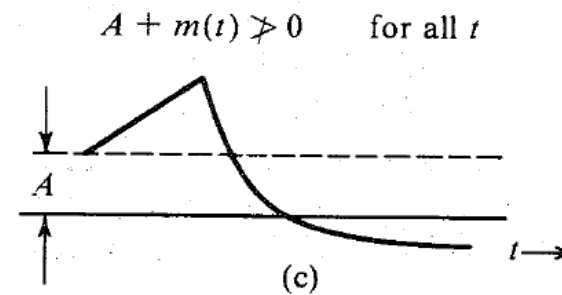
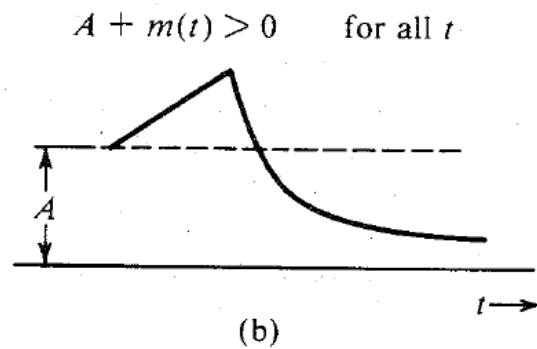
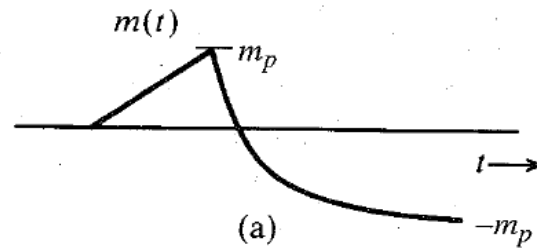


Analog Modulation

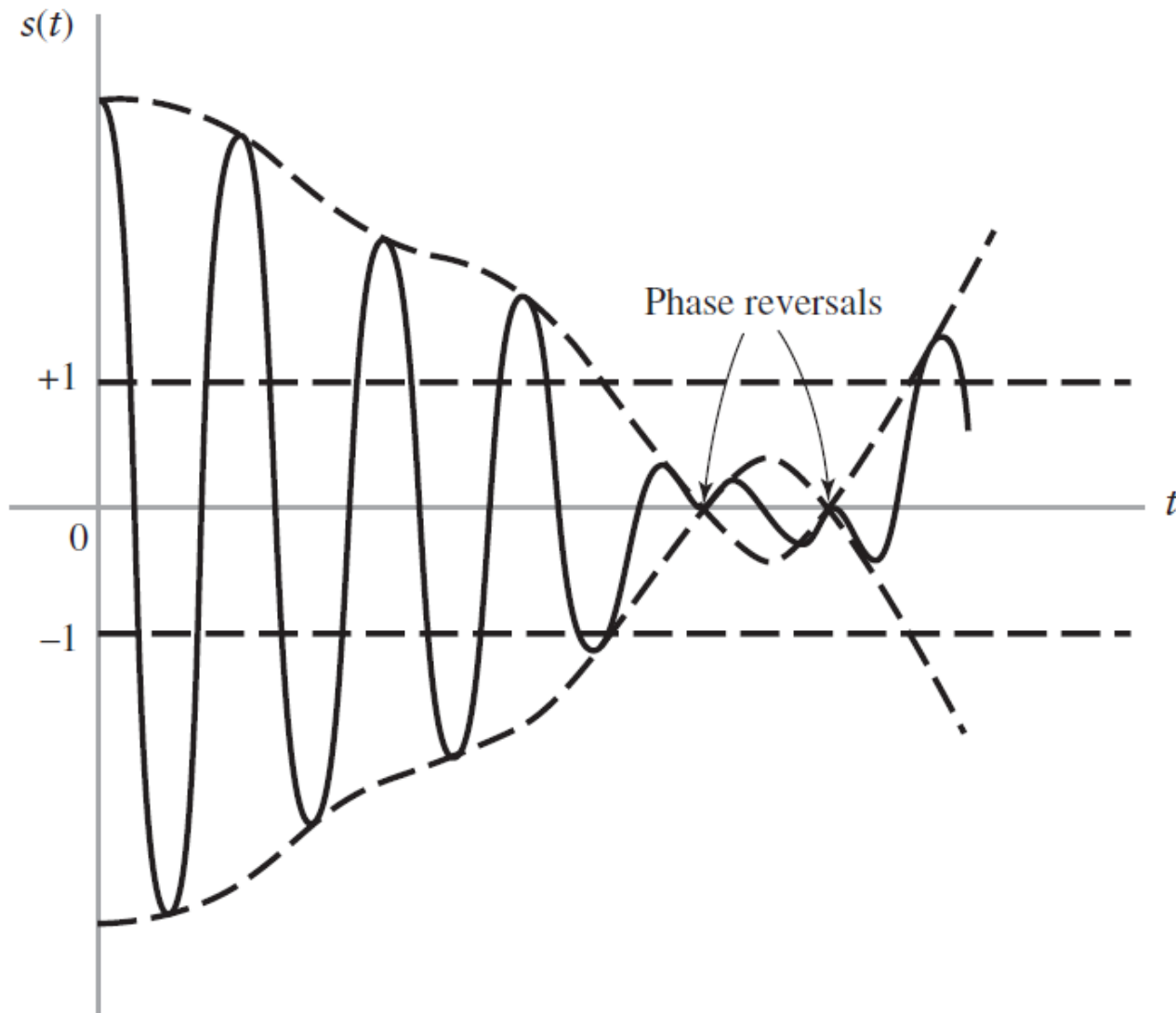
Definition (reminder)

- Message signal $m(t)$: information-bearing signal that is to be recovered at the receiver
- Carrier: the sinusoid with frequency f_c that is used to "carry" the information signal
- Envelope: the time-varying magnitude of the sinusoidal signal
- Modulation factor or index (μ): amount that $m(t)$ is allowed to modulate the carrier

More on DSB-TC

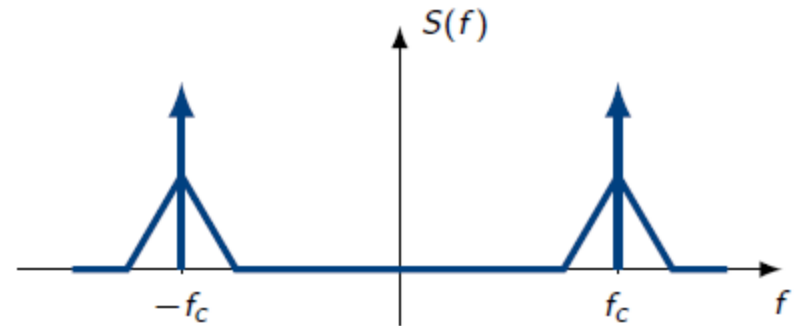
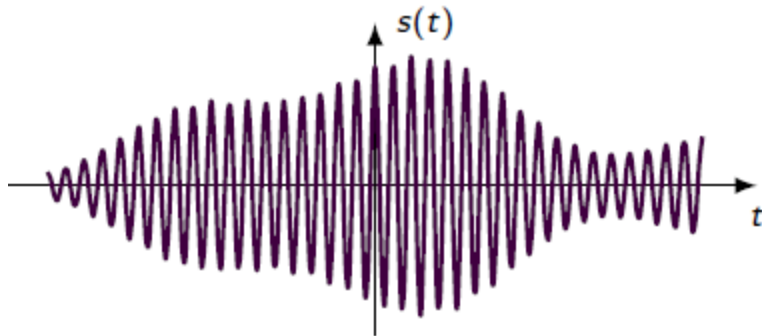
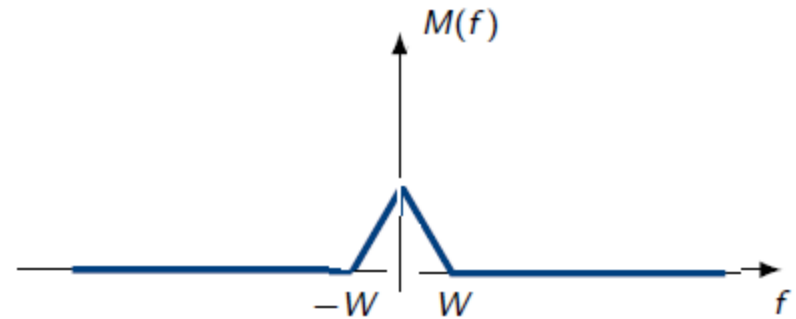
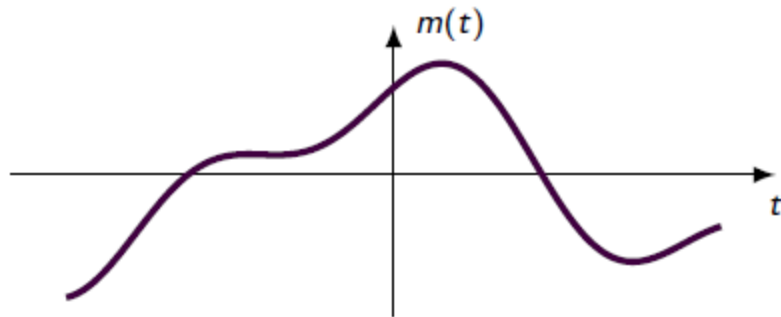


Over-modulation



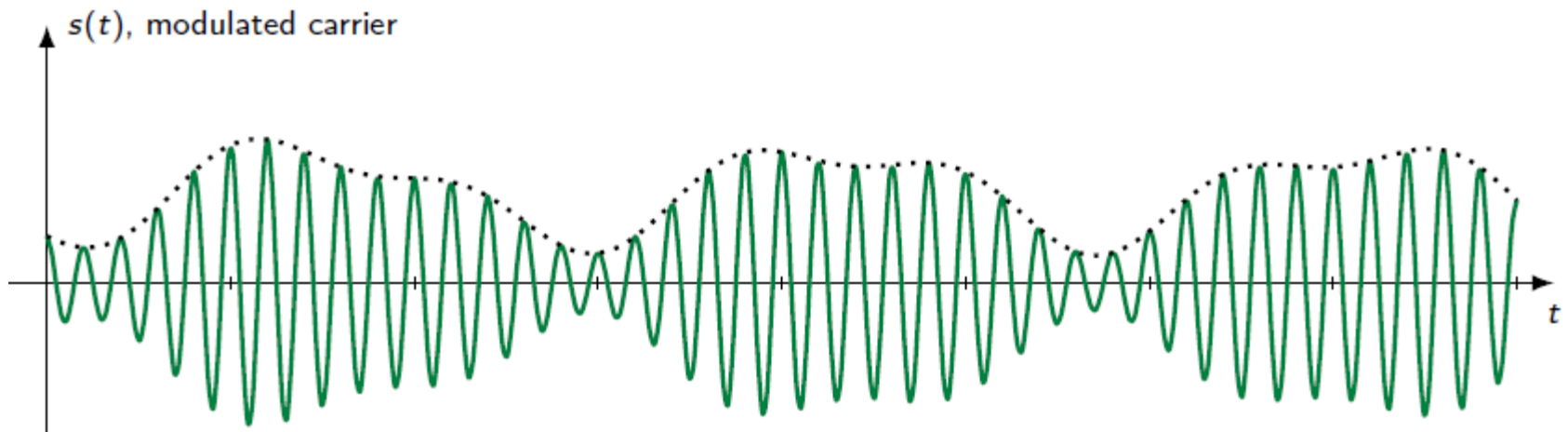
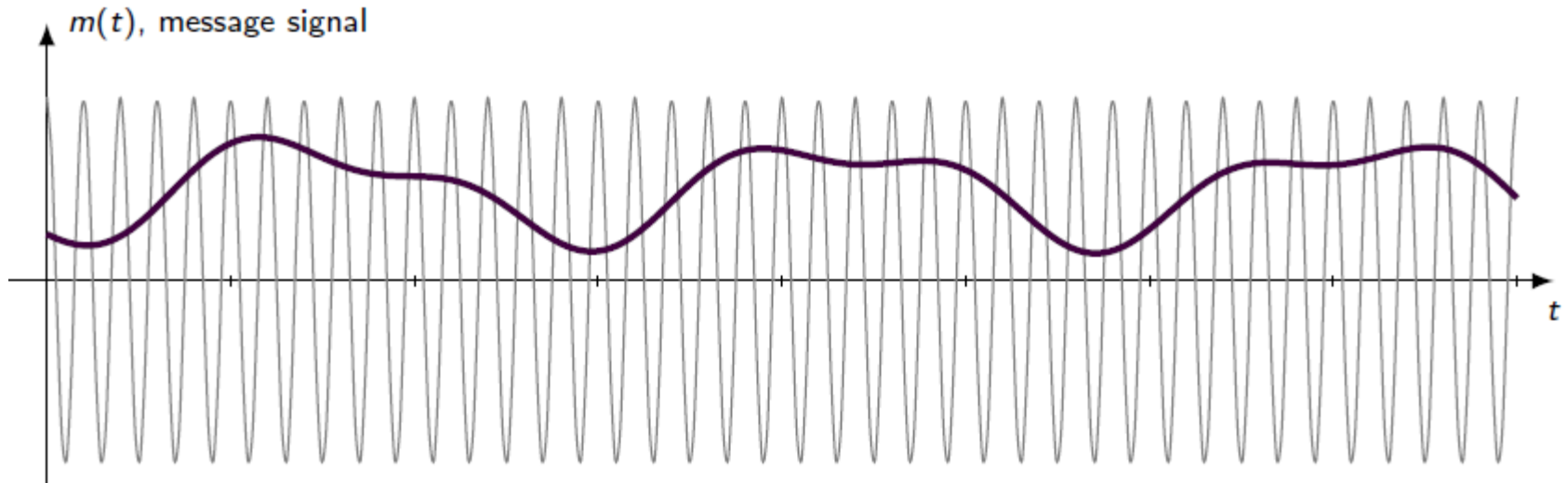
Analog Modulation

Spectrum of DSC-TC AM (illustrative)



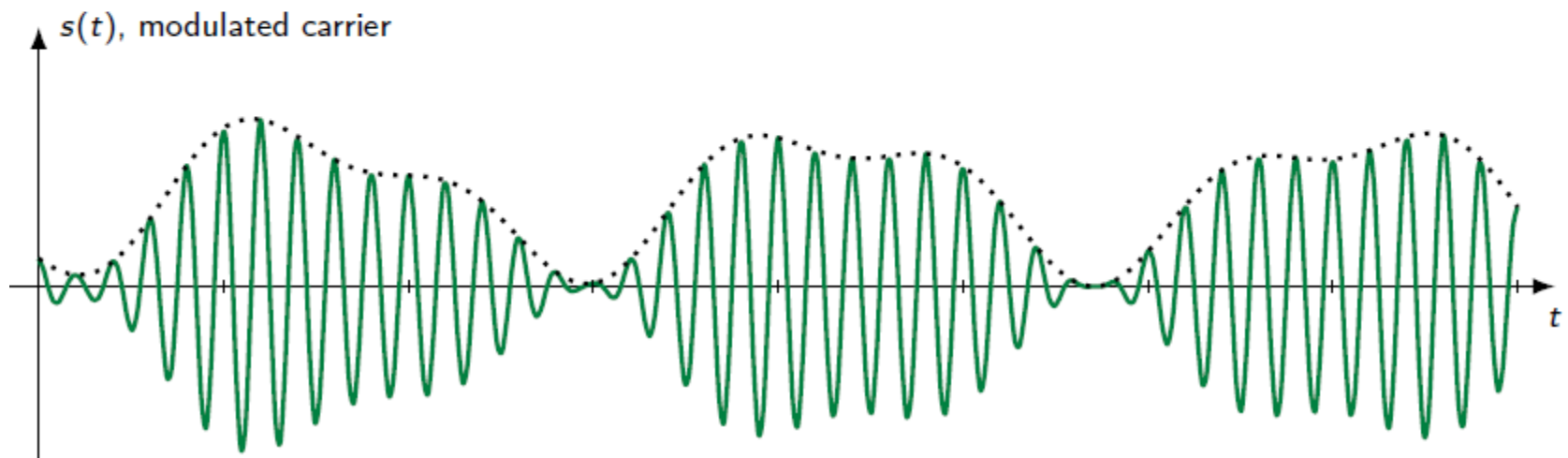
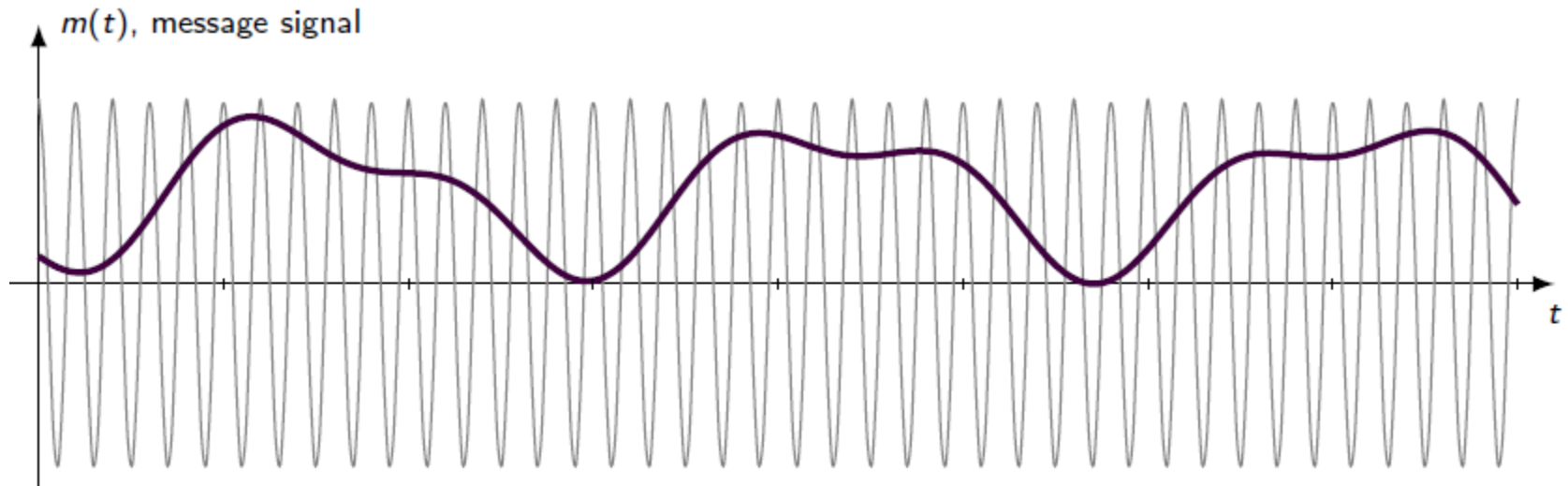
Modulation Examples

DSC-TC AM, $\mu = 0.7$ (typical modulation)



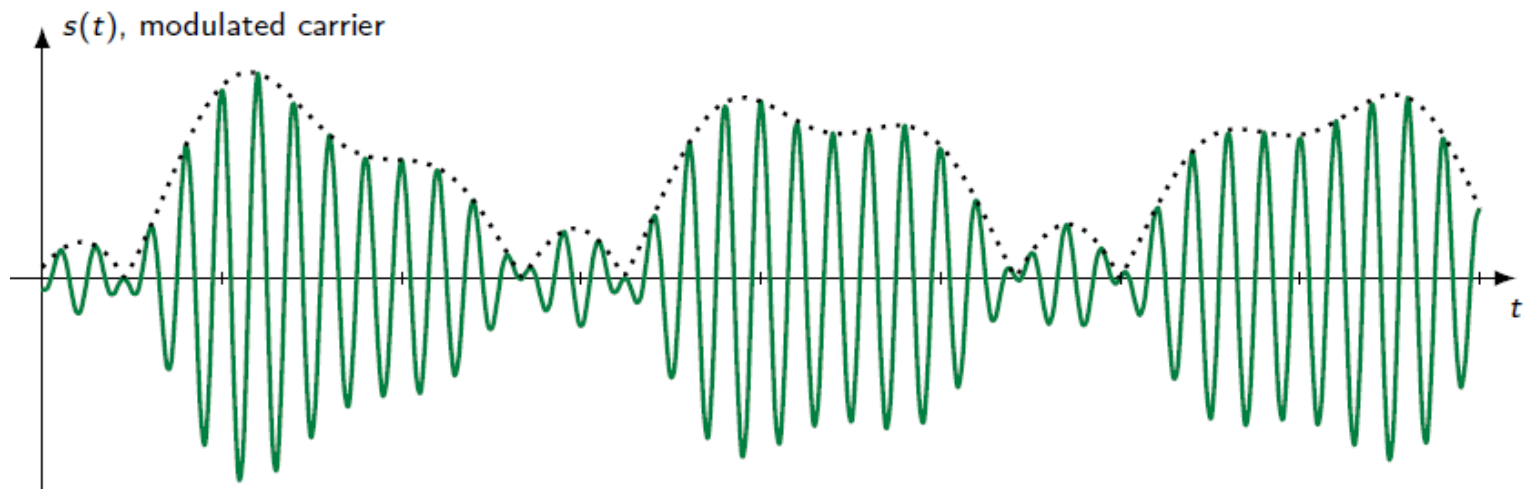
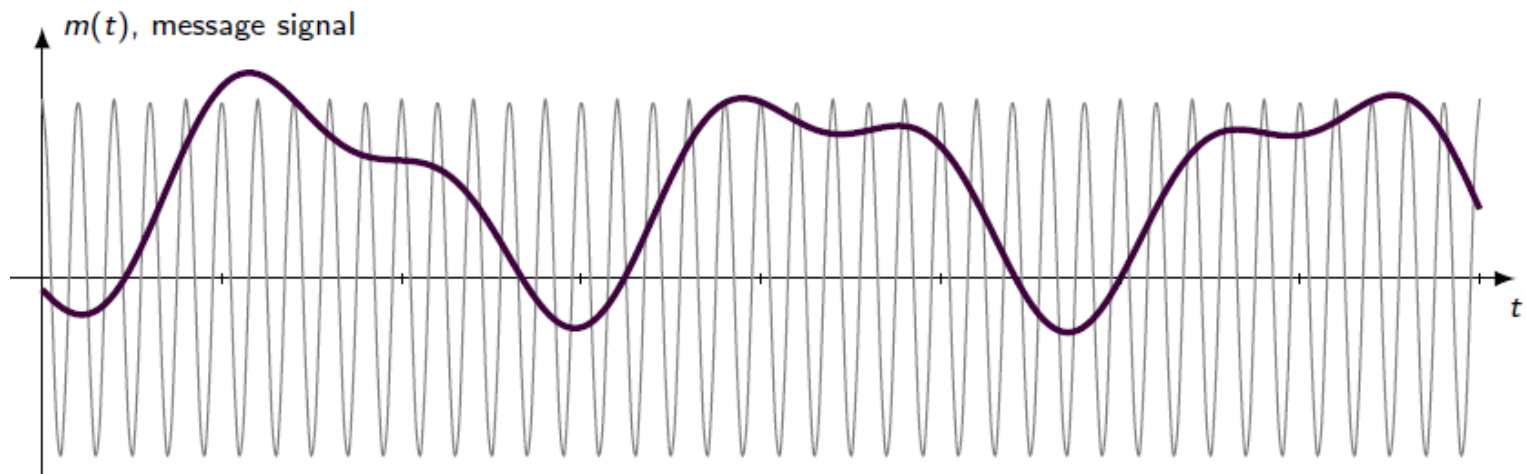
Modulation Examples

DSC-TC AM, $\mu = 1.0$ (maximum modulation)



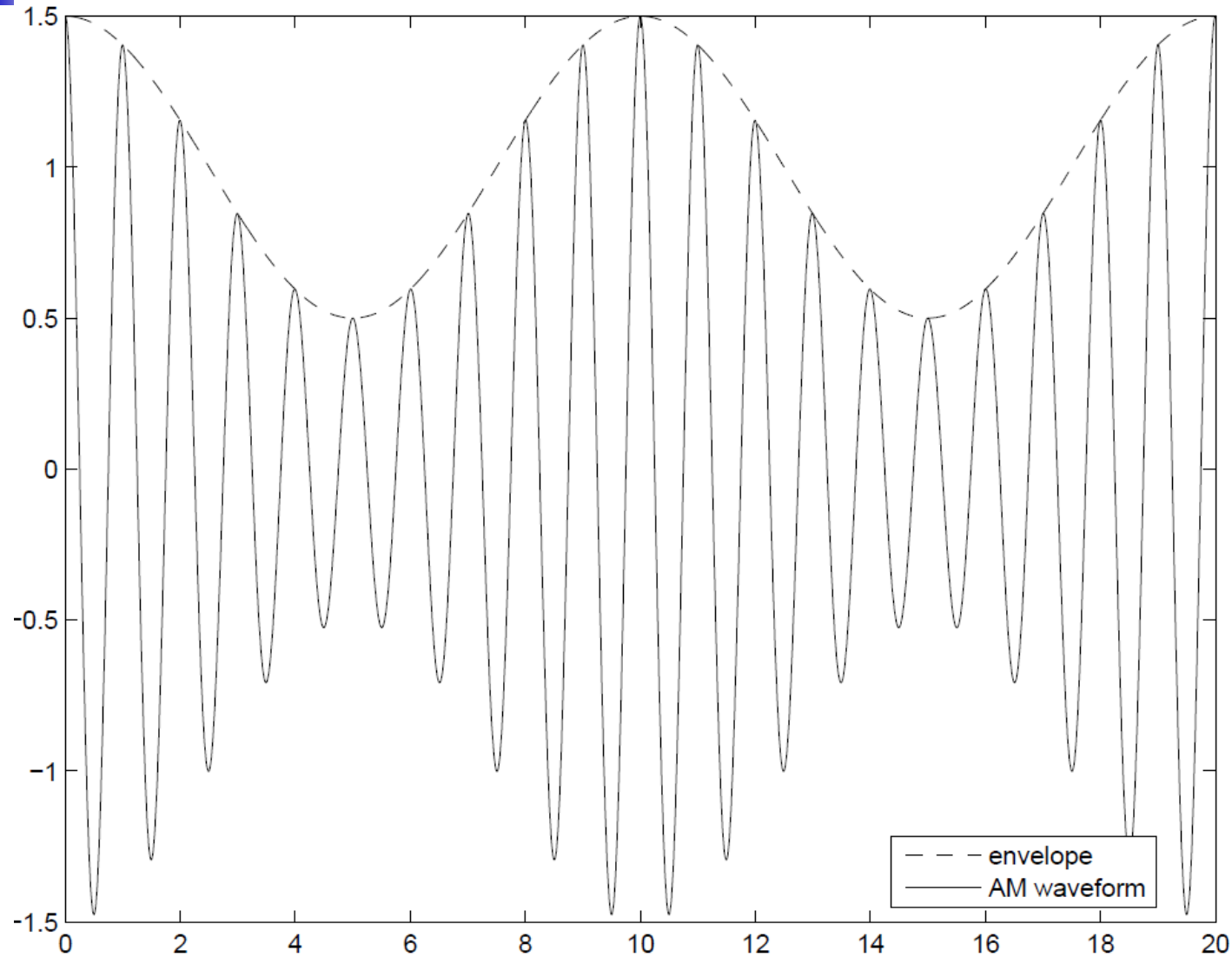
Modulation Examples

DSC-TC AM, $\mu = 1.6$ (over modulation)



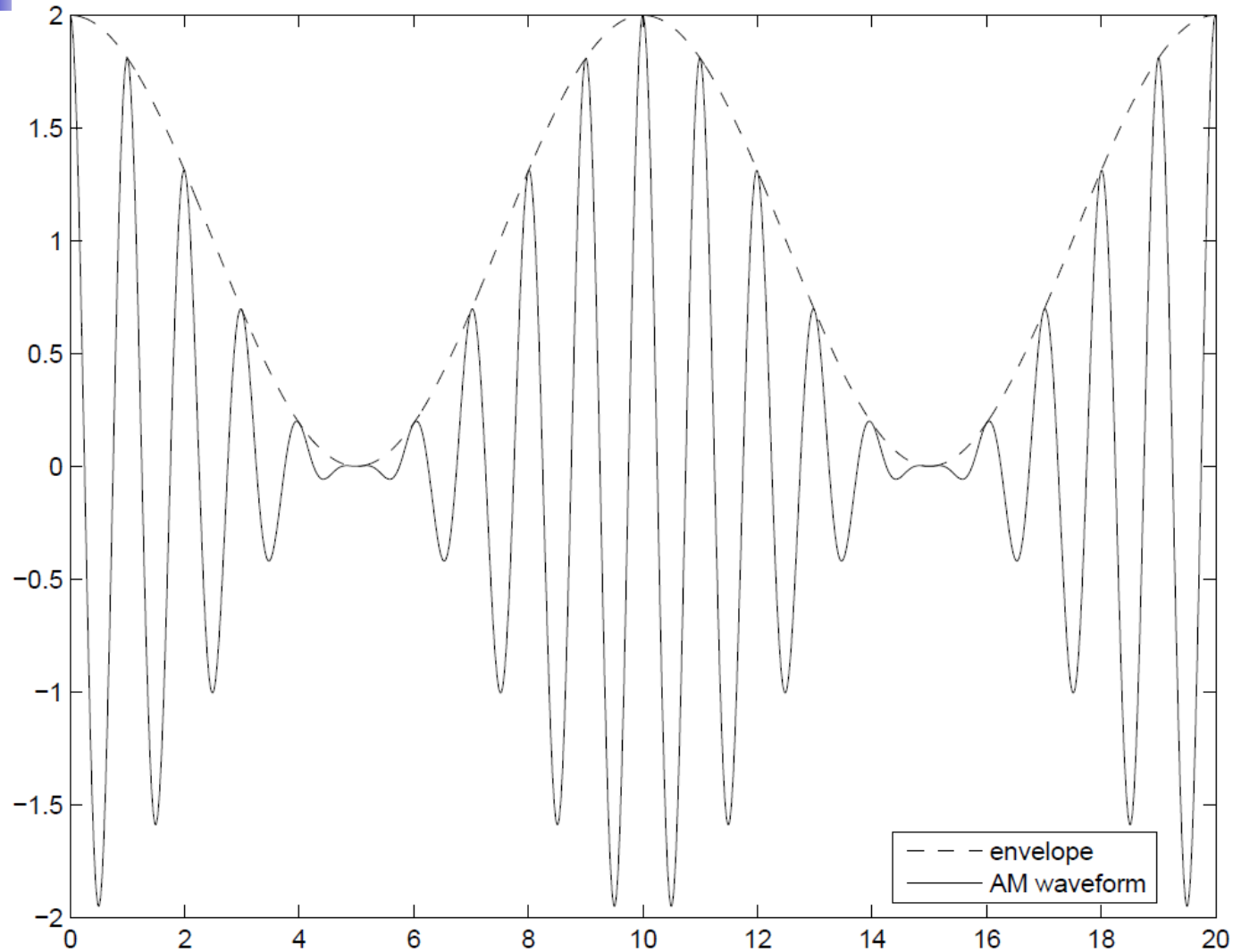
Modulation Examples

DSC-TC AM, $\mu = 0.5$



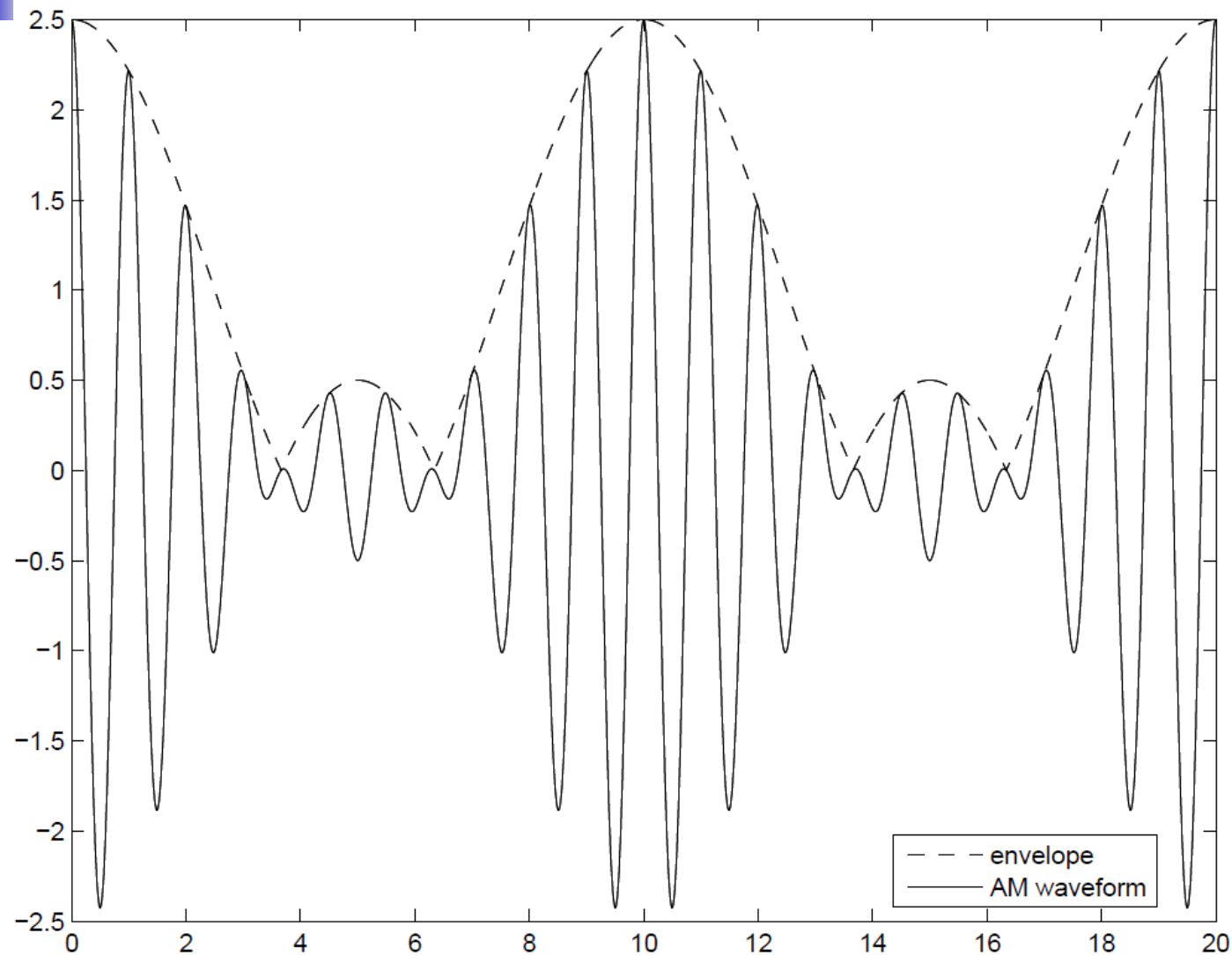
Modulation Examples

DSC-TC AM, $\mu = 1$



Modulation Examples

DSC-TC AM, $\mu = 1.5$ (over-modulation)





Analog Modulation

Double Side-band Transmitted Carrier AM (DSB-TC AM)

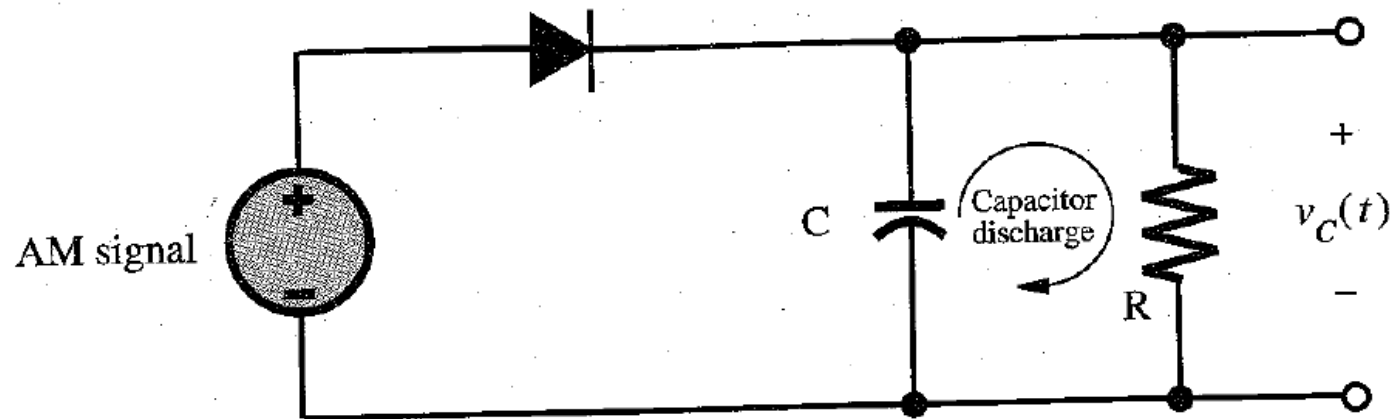
- Another representation used in some references
- Equivalent to before, but slightly different
- Transmitted signal:

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

- k_a is called amplitude sensitivity, or even at times the modulation index (see below)
- Now we require $|k_a m(t)| < 1$
- When this is violated we have "over-modulation"
- Modulation index is now (using past definition): $\max\{|k_a m(t)|\}$
- In some sense, k_a is now acting like the modulation index

Amplitude Modulation

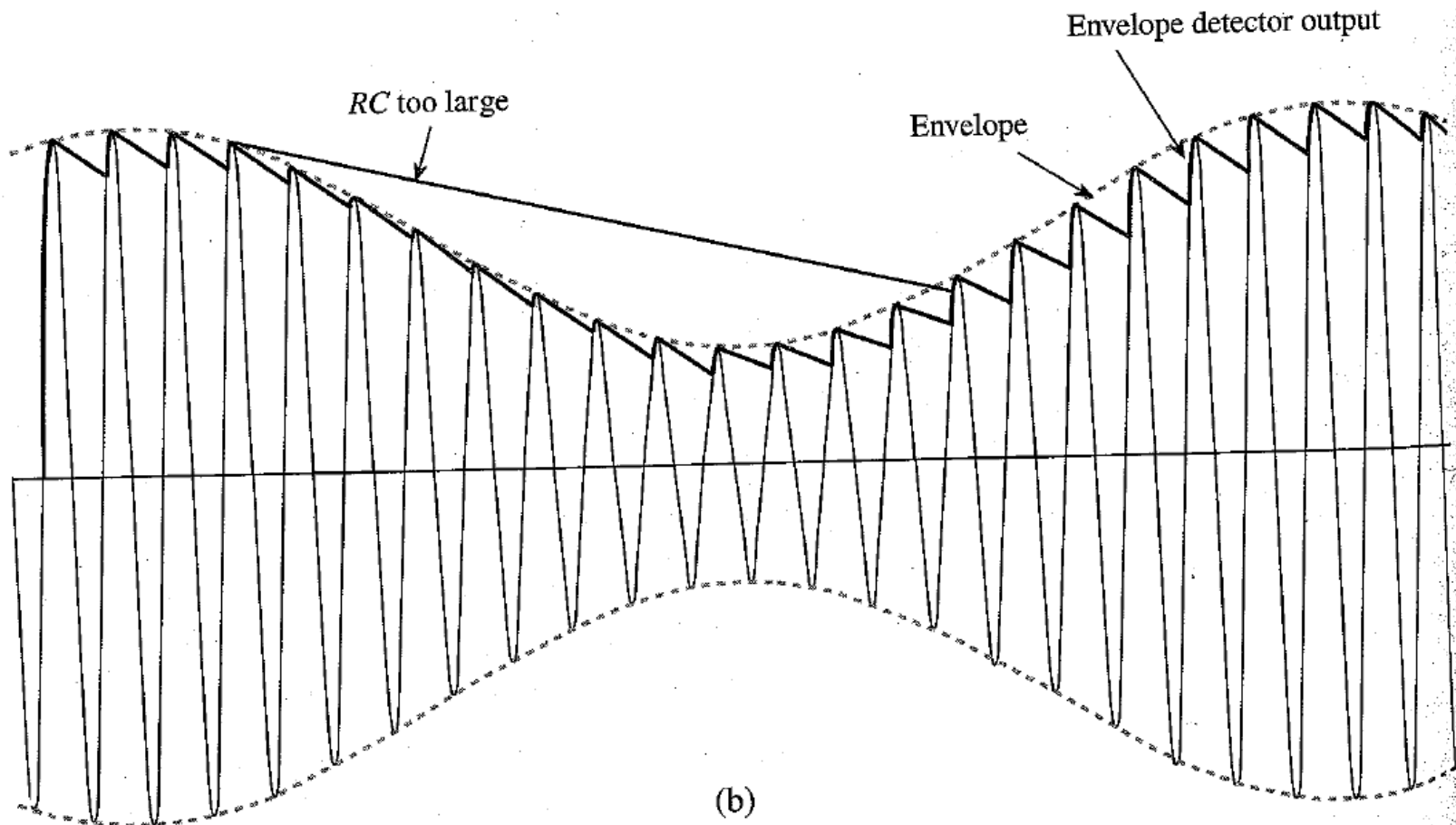
Demodulating DSB-TC AM with Envelope Detector



- Envelope detector is a non-linear system but a very simple receiver to implement
- The output of the detector follows the envelope of the modulated signal
- Uses single diode to only allow positive values through

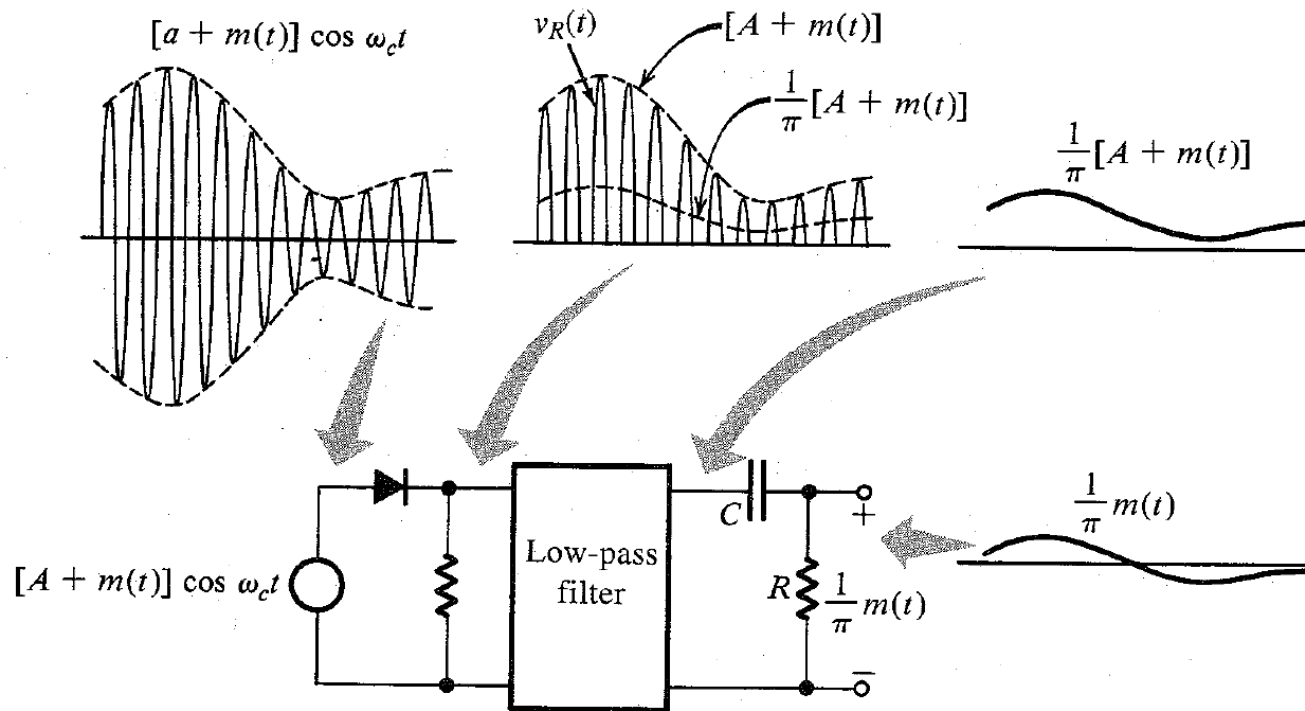
Amplitude Modulation

Demodulating DSB-TC AM with Envelope Detector



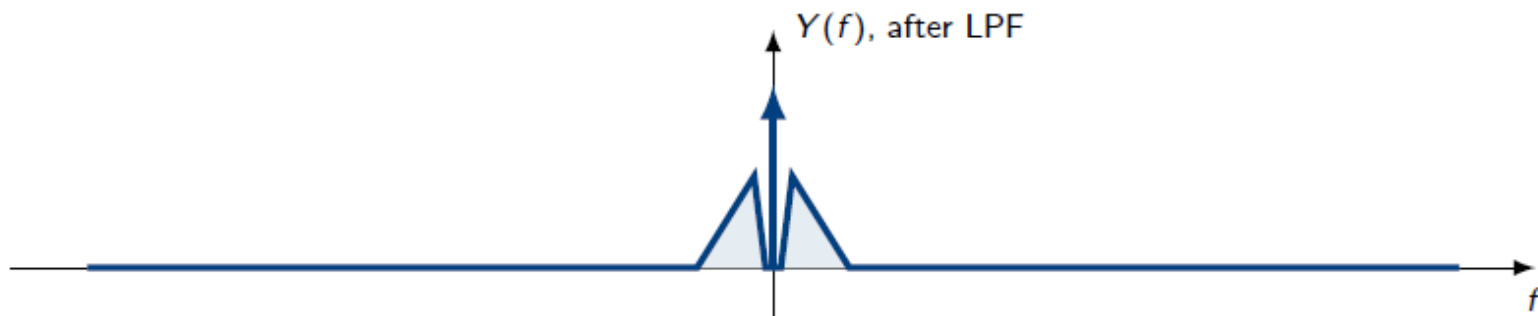
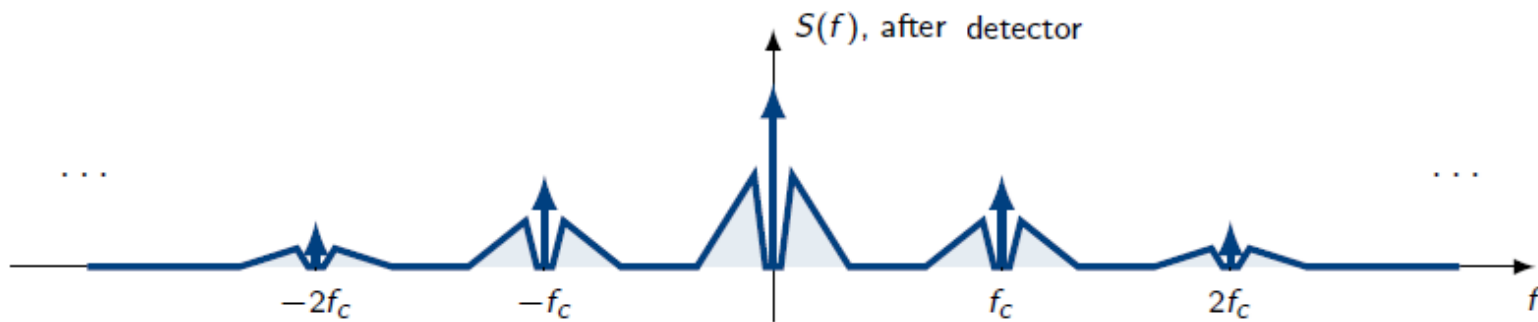
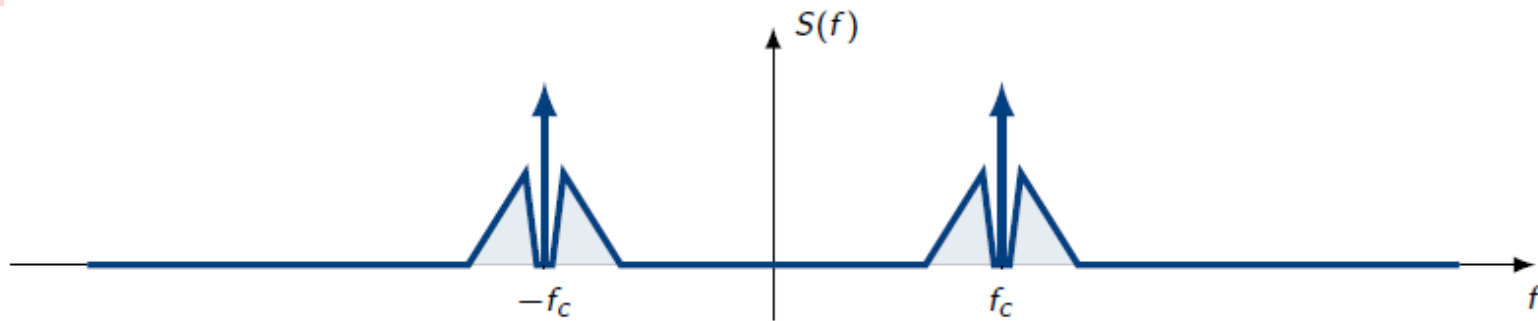
Amplitude Modulation

Demodulating DSB-TC AM with Rectifier



- Half-wave rectifier with low-pass filter
- Negative part of the AM wave is first removed, yielding a half-wave rectified version of the AM signal

Double side-band transmitted carrier AM Spectrum



Double side-band transmitted carrier AM

A little history

■ Foxhole radio

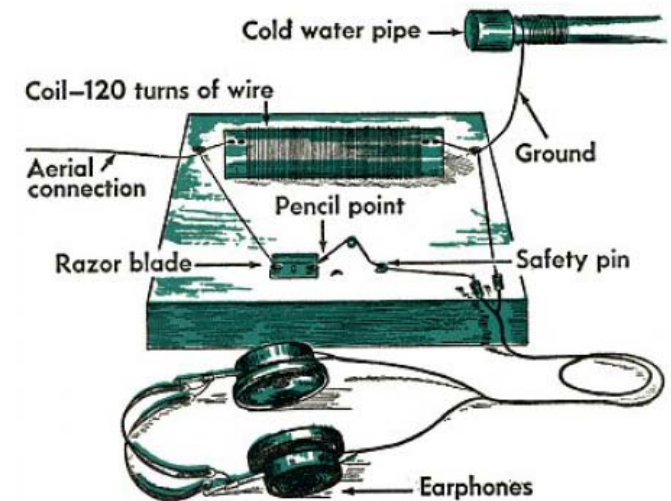
- Prisoners of War (POW) in World War II would build these
- Able to demodulate and listen to AM radio
- No active components

■ Materials

- Razor blade, any wire to form a coil
- Lead from a wood pencil
- Headphones (2-4 k Ω)

■ Functional description

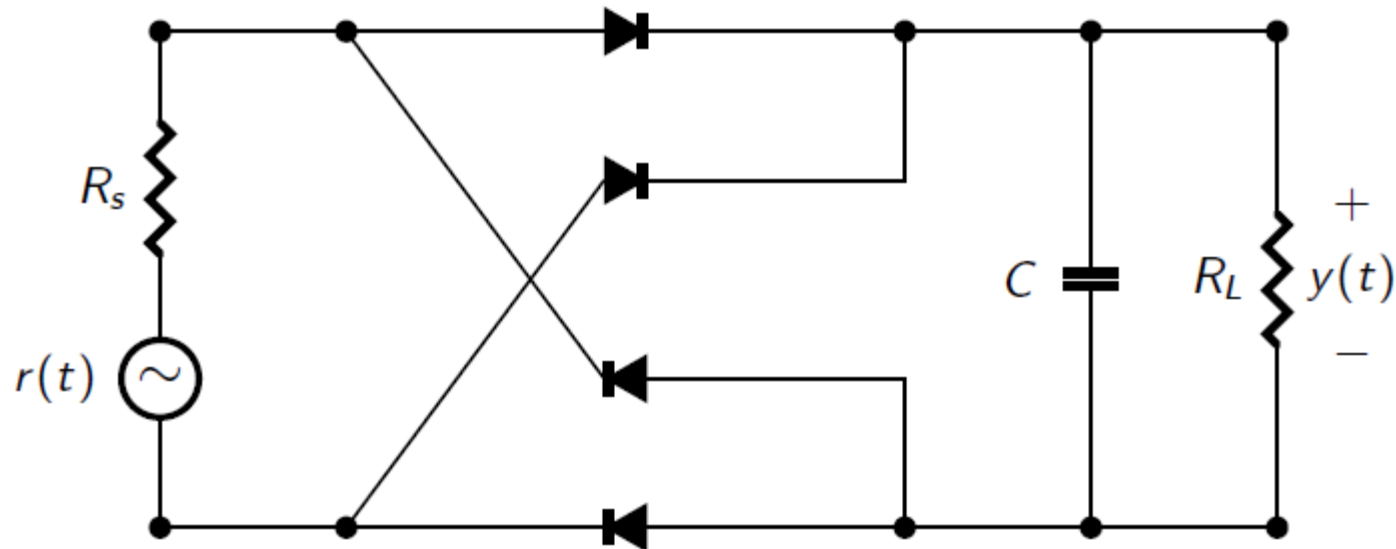
- coil of wire used to tune frequency
- razor blade and pencil lead acted as diode
- Headphones' coil acted as low-pass filter (and speaker)



Foxhole radio (www.solder.net)

Amplitude Modulation

Demodulating DSB-TC AM with Full Wave Rectifier



- Full-wave rectifier with low-pass filter (flips the negative wave into positive wave)
- Fast charge through forward-biased diode
- Slow discharge through load, R_L
- More efficient than half-wave (but slightly more complicated)



Amplitude Modulation

Example

- Sketch the magnitude spectrum of a DSB-TC AM signal $s(t)$ where the message signal is $m(t) = 10/3 \text{sinc}(10t)$ with a carrier frequency $f_c = 20$ Hz.

Amplitude Modulation

Example

- Solution:

$$\begin{aligned} M(f) &= \frac{1}{3} \mathcal{F}\{10\text{sinc}(10t)\} \\ &= \frac{1}{3} \Pi(f/10) \end{aligned}$$





Amplitude Modulation

Tone Modulation

- Recall the transmitted signal for DSB-TC AM:

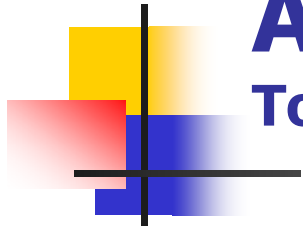
$$s(t) = A_c \cos(2\pi f_c t) + m(t) \cos(2\pi f_c t)$$

- If the message is a sinusoid $m(t) = b \cos(\omega_m t)$, we have the min and max of $m(t)$ equal to $-b$ and b , respectively
- The modulation index is now given by:

$$\mu = \frac{b}{A_c} \quad b = \mu A_c$$

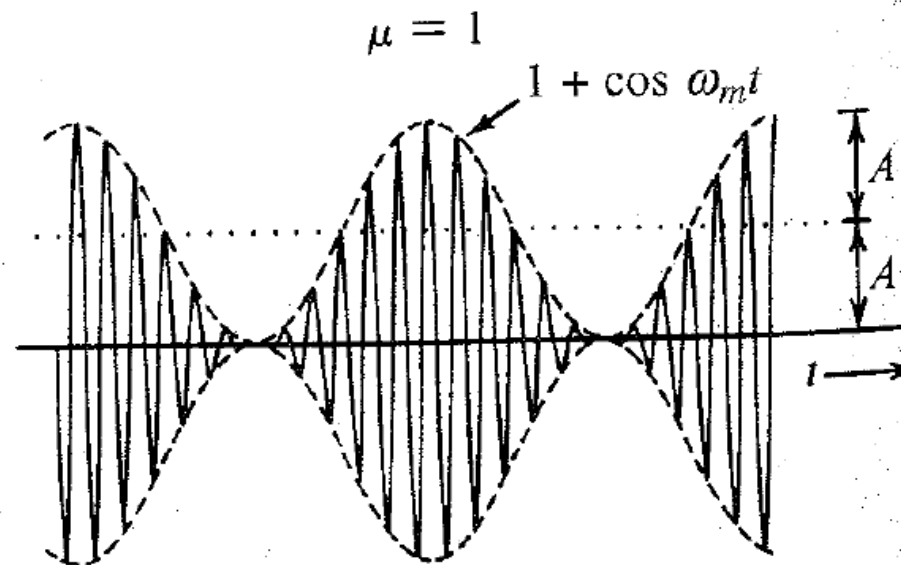
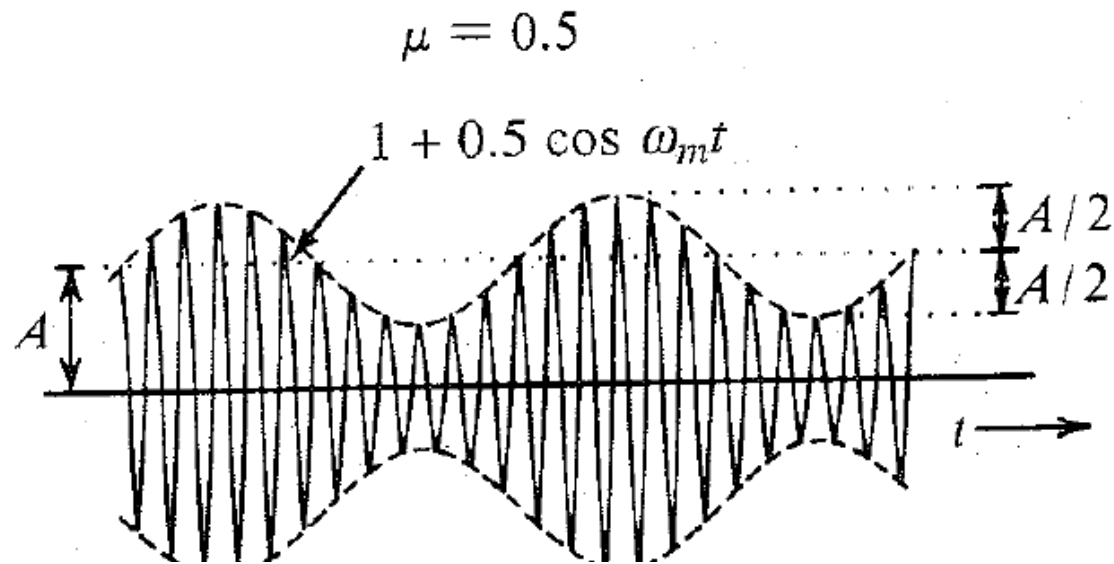
- Therefore, we can now write the modulated signal as:

$$s(t) = [A_c + m(t)] \cos(\omega_c t) = A_c [1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$$



Amplitude Modulation

Tone Modulation





Amplitude Modulation

Power efficiency of DSB-TC AM

- Sideband and carrier power, recall that we have

$$s(t) = A_c \cos(2\pi f_c t) + m(t) \cos(2\pi f_c t)$$

- The first term is the carrier and the second term are the sidebands

- The power of the carrier P_c is given by $A_c^2/2$
- Recall from our PSD discussion that we can write the power of the sidebands as

$$P_s = \frac{1}{2} P_m$$

- We now define power efficiency of DSB-TC AM as:

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_s + P_c} = \frac{P_m}{A_c^2 + P_m} \times 100\%$$



Amplitude Modulation

Power efficiency of a tone modulation

- For the special case of tone modulation we have:

$$m(t) = \mu A_c \cos(\omega_m t) \qquad P_m = \frac{(\mu A)^2}{2}$$

- The power efficiency is therefore:

$$\eta = \frac{\mu^2}{2 + \mu^2} \times 100\%$$

- For $\mu = 0.5$ only 11% of the total power is devoted to the message
- For $\mu = 0.3$ only 4.3 % of the total power is devoted to the message
- For $\mu = 1$ (full modulation), the max of 33% is dedicated to the message/sidebands